

(Time: 3 hours)

Max Marks: 80

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve  $(2x^2 + 3y^2 - 7)x dx + (3x^2 + 2y^2 - 8)y dy = 0$  5

b) Solve  $(D^2 - 4D + 4)y = e^{2x} + \cos 3x$ , where  $D \equiv \frac{d}{dx}$  5

c) Evaluate  $\int_0^{\infty} x^3 e^{-4x^2} dx$  5

d) Change the order of integration  $I = \int_0^a \int_{-a+\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} f(x,y) dx dy$  5

Q2.

a) Evaluate  $I = \iiint \frac{z^2 dx dy dz}{x^2 + y^2 + z^2}$  over the volume of the sphere 6

$$x^2 + y^2 + z^2 = 2$$

b) Find the length of the cardioid  $r = a(1 + \cos \theta)$  which lies outside the circle  $r + a \cos \theta = 0$  6

c) Solve  $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$  by using the method of Variation of parameters. 8

Q3.

a) Prove that  $\int_0^{\infty} \frac{1-\cos ax}{x} e^{-x} dx = \frac{1}{2} \log(1+a^2)$ , assuming the validity of differentiation under the integral sign. 6

b) Evaluate  $I = \iint y dx dy$  over the area bounded by 6

$$x = 0, y = x^2, x + y = 2 \text{ in the first quadrant.}$$

c) Evaluate the integral 8

$$I = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dx dy dz.$$

**Q4.**

a) Solve  $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$  **6**

b) Solve  $(D^2 + 3D + 2)y = e^{e^x}$  **6**

c) Prove that  $\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} \cdot \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}$  **8**

**Q5.**

a) Change the integral to polar coordinate and evaluate **6**

$$I = \int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dy dx$$

b) Find area of one loop of the lemniscate  $r^2 = a^2 \cos 2\theta$  **6**

c) Solve  $\frac{dx}{dy} - xy = x^3 y^3$  **8**

**Q6.**

a) Solve  $(D^2 - 4)y = x \sinh x$  **6**

b) Solve  $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$  **6**

c) Change the order of integration and evaluate **8**

$$I = \int_0^a \int_y^{\sqrt{ay}} \frac{x}{x^2+y^2} dx dy$$

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